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# Shape Matching Part 0: Intro

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# Why shape

- User interviews show:  
for object retrieval, shape is more important than color and texture
- Problem: effective shape is more difficult than effective color and texture
- Check yourself: e.g. with Blobworld:

<http://elib.cs.berkeley.edu/photos/blobworld/start.html>

# Shape Algorithmics

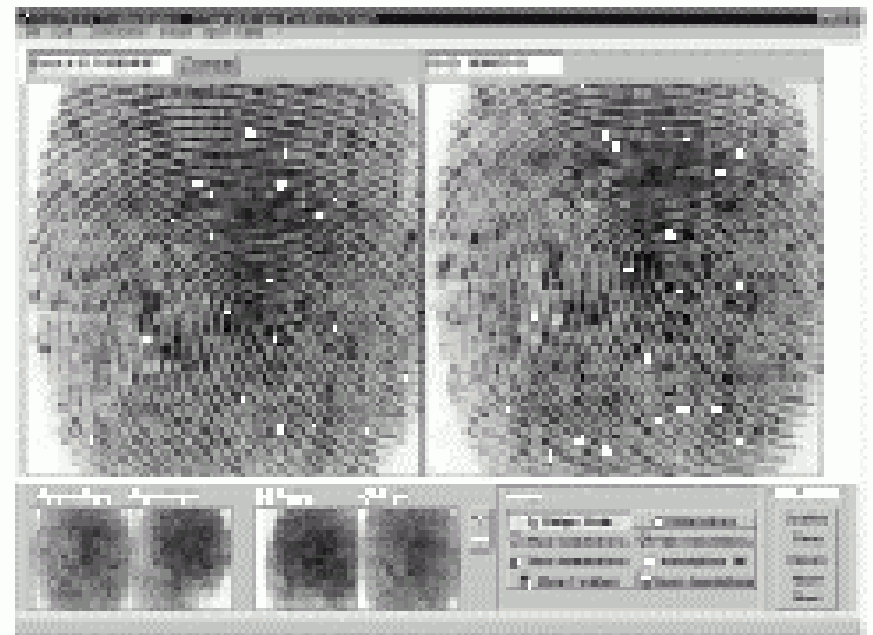
- Algorithmic aspects of shape analysis:  
points, curves, regions, volumes, all representing  
form (shape)
- Aspects:
  - representation, decomposition, approximation,  
deformation of shape
  - transformation of one shape onto another
  - measuring how similar two shapes are
  - organization of shapes in search structures

# Shape Matching

- Given:
  - two shapes  $A, B$
  - distance function (or dissimilarity measure)  $d$
  - transformation  $g$  or group  $G$
- Compute or minimize  $d(g(A), B)$

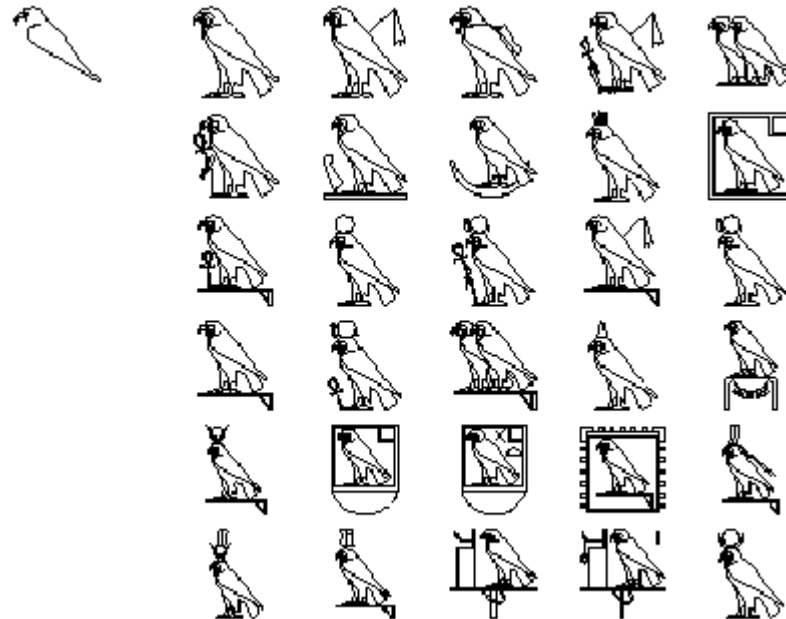
# Typical problems

- What is the matching transformation?
- No one-to-one correspondence
- Occlusion
- Noise



# Typical problems

- Partial match: only part of query appears in part of database shape



# Which algorithm?

Depends on

- which similarity measure, depends on
- which required properties, depends on
- which particular matching problem, depends on
- which application

# Which application?

- Retrieval
- Recognition and classification
- Alignment, registration
- Approximation

# Which problem?

- Computation problem:  $d(A,B)$
- Decision problem:  $d(A,B) \leq \varepsilon$  ?
- Decision problem: is there  $g: d(g(A),B) \leq \varepsilon$  ?
- Optimization problem:  
find  $g^* = \operatorname{argmin}_{g \in G} d(g(A),B)$
- Approximate optimization problem:  
find  $g: d(g(A),B) < k d(g^*(A),B)$ , for some fixed  $k$

# Which properties?

- Metric properties
- Continuity
- Invariance

# Metric Properties

- $S$  set of patterns
- Metric:  $d: S \times S \rightarrow R$  satisfying
  1. Self-identity:  $\forall x \in S, d(x,x)=0$
  2. Positivity:  $\forall x \neq y \in S, d(x,y) > 0$
  3. Symmetry:  $\forall x, y \in S, d(x,y) = d(y,x)$
  4. Triangle inequality:  $\forall x, y, z \in S, d(x,z) \leq d(x,y) + d(y,z)$
- Semi-metric: 1, 2, 3
- Pseudo-metric: 1, 3, 4
- $S$  with fixed metric  $d$  is called metric space

# Symmetry

$$d(A,B) = d(B,A)$$

not always so for human perception

variant A:



prototype B:

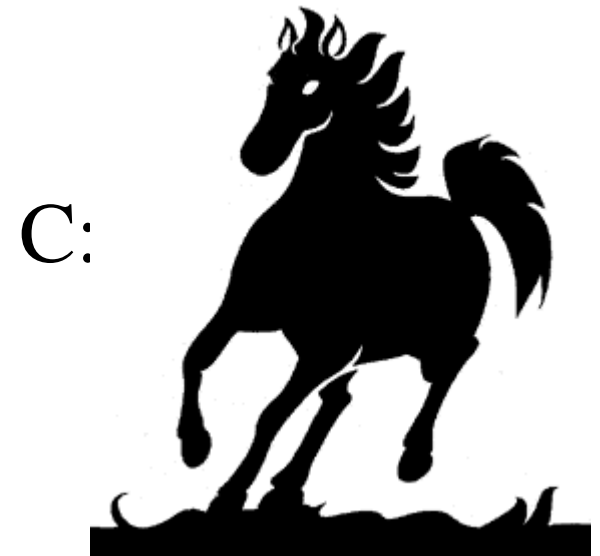
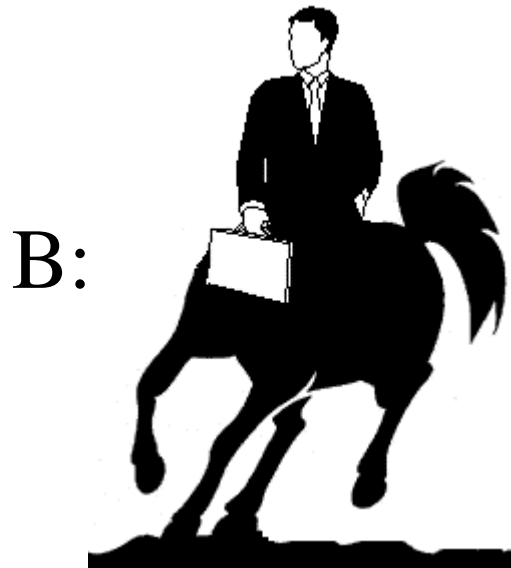
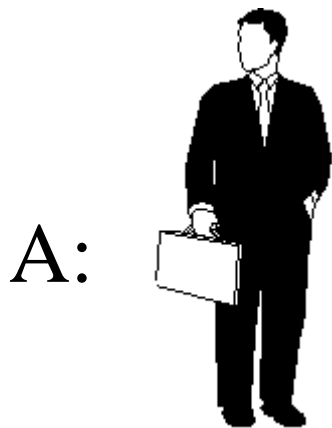


$$d(A,B) < d(B,A)$$

# Triangle inequality

$$d(A,B)+d(B,C) \geq d(A,C)$$

not always so for human perception,  
in particular for partial matching:

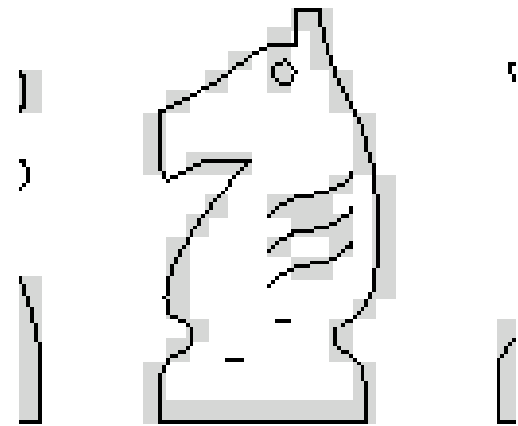


# Continuity

## Robustness

arbitrary small changes:

- deformation
- blurring
- cracks
- noise



lead to arbitrary small change in similarity

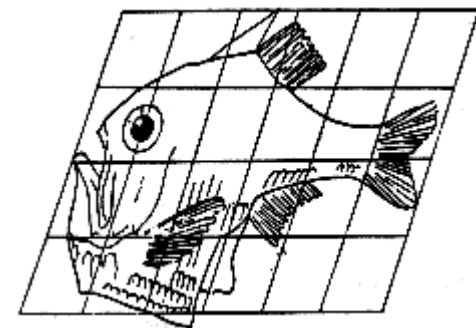
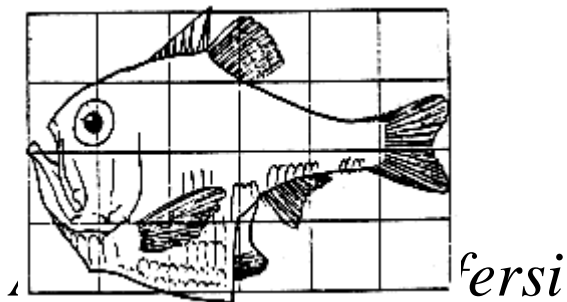
# Invariance

$$d(g(A),g(B)) = d(A,B)$$

for all  $g$  in transformation group



$$d(g(A),B) = d(A,B)$$



*dialphana*

(D'Arcy Thompson, 1911)


# Invariance?

Content-based Image Retrieval System on A Fish Database of Taiwan - Academia Sinica, Taiwan - Netsca...


File Edit View Go Communicator Help

Bookmarks Location: [http://smart.iis.sinica.edu.tw/~fish/eep\\_demo.html](http://smart.iis.sinica.edu.tw/~fish/eep_demo.html) What's Related

[Back](#) Images are randomly selected on a fish image database of TAIWAN [Other samples](#)







**Fish body image**



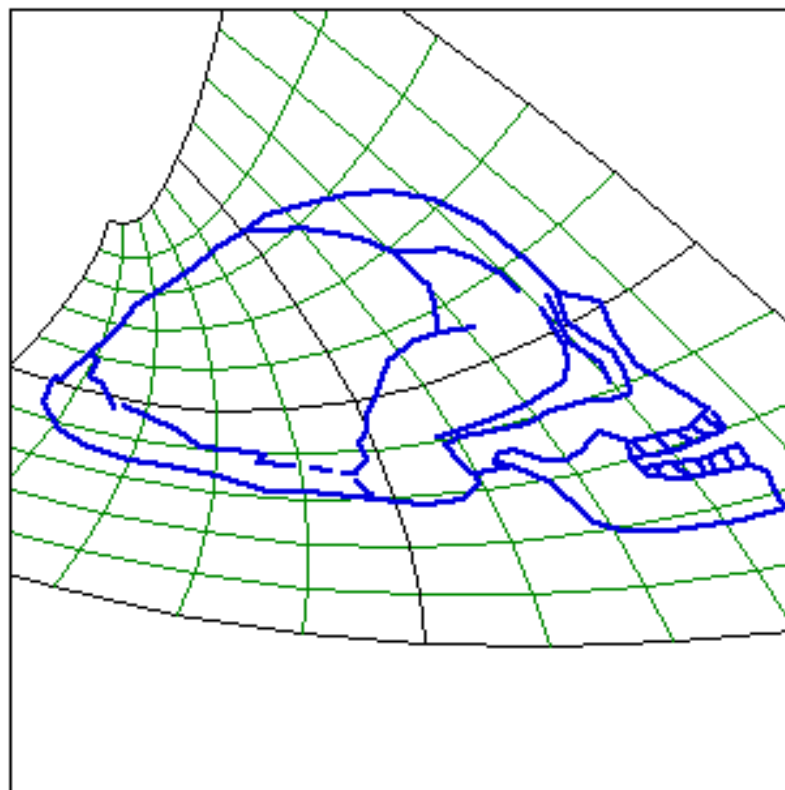
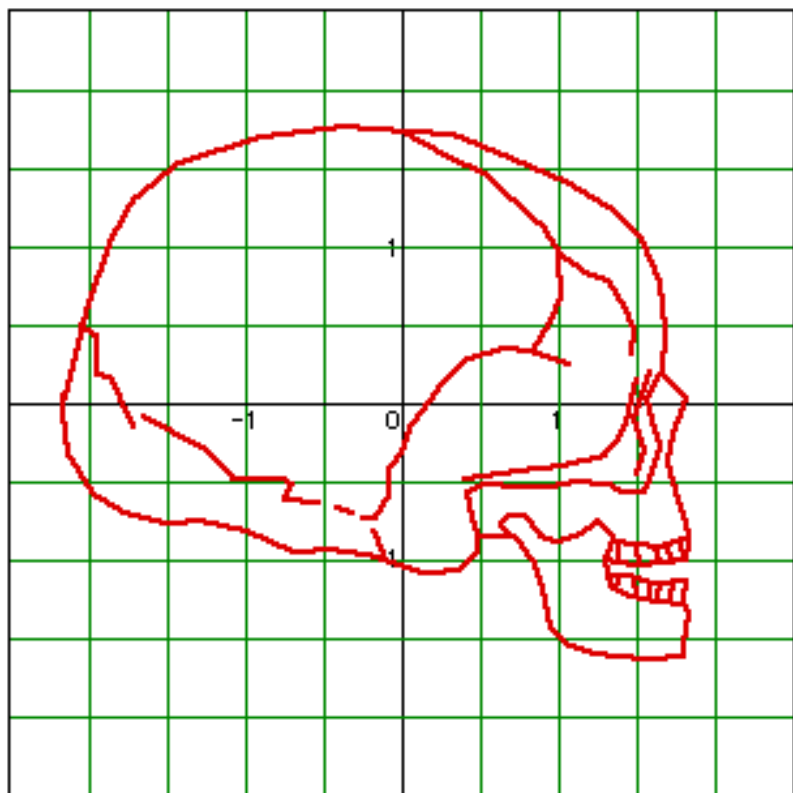
Query

**Query Results**

Image	Species
	Hypophthalmichthys molitrix
	Cheilodipterus quinquelineatus
	Aristichthys nobilis
	Oncorhynchus masou formosanus

# Invariance?

for large enough transformation group ...



# Which similarity?

discrete metric:

$$d(A,B) = \begin{cases} 0 & \text{if A equals B} \\ 1 & \text{otherwise} \end{cases}$$

exact congruence matching

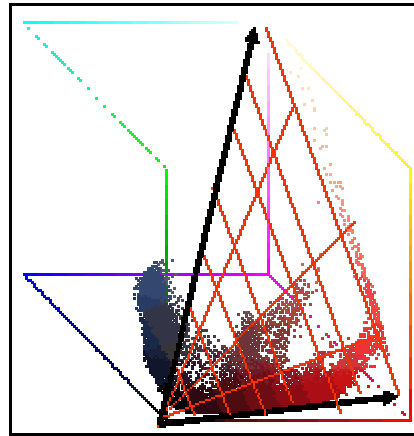
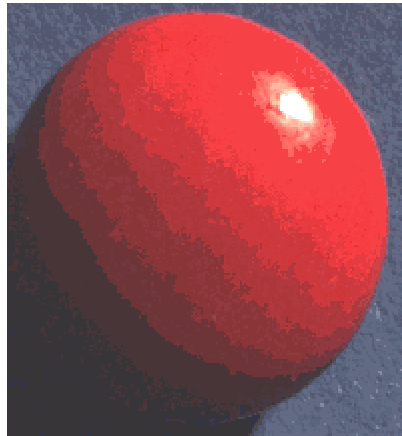
metric, invariant under all homeomorphisms!

but lacks robustness properties

# What is shape anyway?

Plato, "Meno", 380 BC:

*"figure is the only existing thing that is found  
always following color"*



What is shape?

"terms employed in geometrical problems":

*"figure is limit of solid"*



# Levels of Shape

- Two orthogonal categorizations:
- Global/local shape
- Image/object shape

# Global, Image

- Fourier transform
- Discrete cosine transform (used in jpeg)
- Wavelet transform



# Global, Image

- Transformation from color in spatial domain to color variation in frequency domain
- No explicit shape encoding, but intensity transitions at object boundaries
- Because whole image is encoded, no matching of individual shapes

# Types of Object Shape

- Point pattern
- Curve
- Contour
- Region

# Global, Object

- Matching of a single, whole, object
- Features:
  - Elementary descriptors
  - Moments
  - Modal matching
  - Fourier descriptors
  - Curvature scale space
- Drawbacks:
  - Relies on perfect segmentation
  - Sensitive to noise and occlusion

# Local, Object

- Matching local descriptors
- Weak segmentation
- Features:
  - Corners, relative position
  - Corners, curvature

# Approaches

Two classes of shape matching approaches:

- Based on feature vectors
  - Intermediate step: information loss
  - Simple distance calculations
- Based on direct shape comparisons
  - Complex distance calculations