Hierarchical Modeling with Parametric Surfaces

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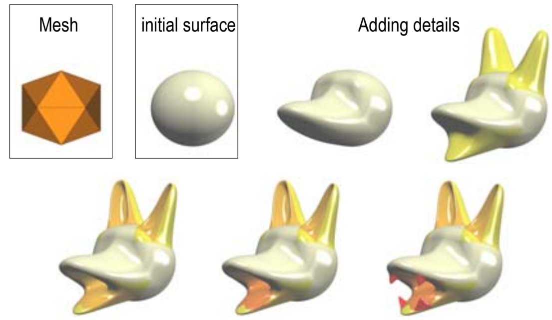
Hierarchical LoD modeling

Construction of levels of detail



Hierarchical Modeling with Parametric Surfaces

Outline :

I. Triangular G1 Surfaces of arbitrary topology

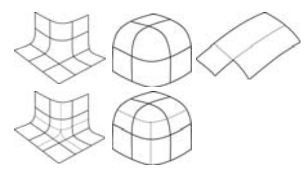
- *I.1 G1 continuity conditions*
- *I.2 4-split method*

II. Hierarchical Triangular Splines

- *II.1 Local refinement*
- *II.2 Hierarchical Edition*
- *II.3 Results*

Why do we prefer triangular patches ?

non-tensor product configurations



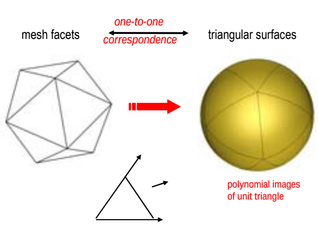
Surfaces of arbitrary topology

Parametric surfaces of arbitrary topology

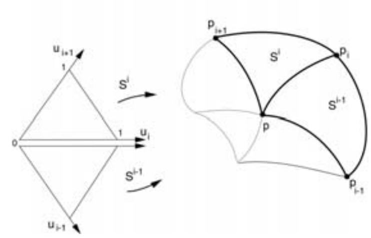
triangular mesh → *smooth surface*

- triangular faces
- arbitrary topology (2d manifold)
- with / without boundary

- mesh interpolation
- parametric, polynomial
- G1 continuous, smooth
- local support
- design parameters
- locally refinable



Parameterization



S^i macro patch (4 triangular Bézier patches)
p mesh vertex
p_i vertex neighborhood, i=1, . . . , n

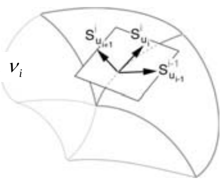
Tangent plane continuity

between adjacent patches :

- common boundary
- existence of 3 scalar functions $\Phi_i, \ \mu_i, \ \nu_i$

continuity constraint

$$\Phi_i \ S^i_{u_i} = \mu_i \ S^{i-1}_{u_{i-1}} + \nu_i \ S^i_{u_{i+1}}$$



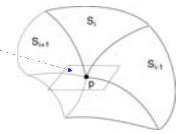
at patch corners :

twist constraint

$$\Phi_i^{'}(0) S^i_{u_i} + \Phi_i(0) S^i_{u_i u_i} = \mu_i^{'}(0) S^{i-1}_{u_{i-1}} + \nu_i^{'}(0) S^{i+1}_{u_{i+1}} + \mu_i(0) S^i_{u_{i+1} u_i} + \nu_i(0) S^i_{u_{i-1} u_i}$$

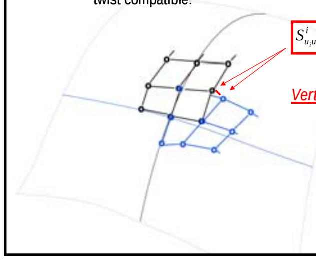
Tangent plane continuity

Vertex common to n patches



- existence of a well defined tangent plane is not sufficient.
- Polynomial patches need to be twist compatible:

$$S^i_{u_i u_{i+1}}(0,0) = S^i_{u_{i+1} u_i}(0,0)$$

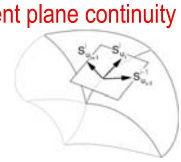


Vertex consistency problem

Tangent plane continuity

continuity constraint

$$\Phi_i \ S^i_{u_i} = \mu_i \ S^{i-1}_{u_{i-1}} + \nu_i \ S^i_{u_{i+1}}$$



twist constraint (in matrix form)

$$(M_3) \boldsymbol{T} = (M_1) \boldsymbol{D}_1 + (M_2) \boldsymbol{D}_2$$

$$D_1^i = \frac{\partial S^i}{\partial u_i}(0,0)$$

$$D_2^i = \frac{\partial^2 S^i}{\partial u_i^2}(0,0)$$

$$T^i = \frac{\partial^2 S^i}{\partial u_i \partial u_{i+1}}(0,0)$$

Vertex consistency problem

matrix (M_3) is rank deficient $\Leftrightarrow$ n is even

Tangent plane continuity

continuity constraint

$$\Phi_i \ S^i_{u_i} = \mu_i \ S^{i-1}_{u_{i-1}} + \nu_i \ S^i_{u_{i+1}}$$

twist constraint

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i=0,...,n

determine

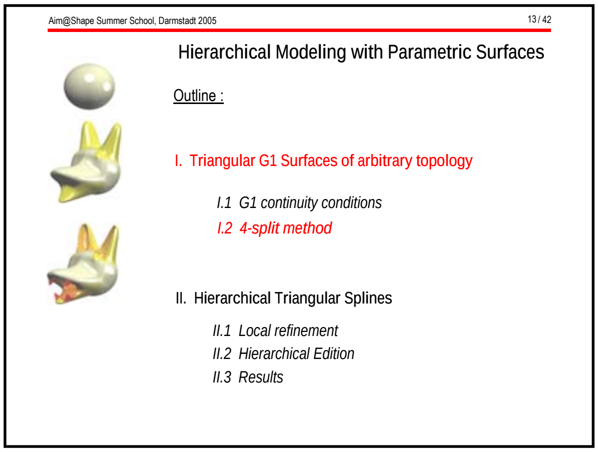
scalar functions Φ_i, μ_i, ν_i
patches S^i, S^{i-1}

⇨ polynomial
⇨ minimal degree
⇨ local definition

Bibliography
on triangular surfaces of arbitrary topology

- G1 conditions at network corners (Van Wijk'86, Du'88)
- Convex combination schemes (Gregory'86, Hagen'86, Nielson'87)
- Clough-Tocher domain splitting (Farin'82, Piper'87, Shirman/Sequin'87)
- Algebraic methods (Bajaj'92)
- Subdivision surfaces (Catmull Clark, Loop, Doo Sabin, ….)
- Surface splines (Peters'91…)
- Manifold splines (Gu, He, Qin 05)
- 4-split methods (Hahmann, Bonneau 00-05)

[3D Studio Max-Plugin realized by Thiele, Oman, Resch Uni Tuebingen, 2001]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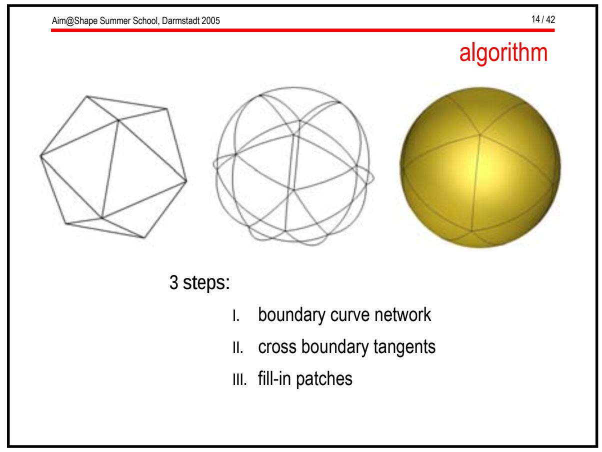
I.1 G1 continuity conditions
I.2 4-split method
II. Hierarchical Triangular Splines
II.1 Local refinement
II.2 Hierarchical Edition
II.3 Results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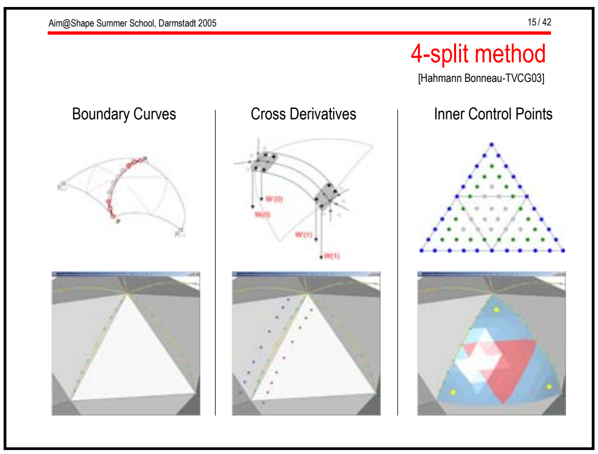
algorithm
3 steps:
I. boundary curve network
II. cross boundary tangents
III. fill-in patches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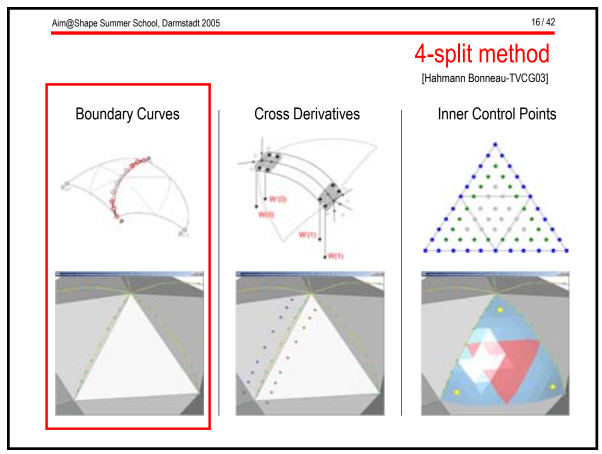
4-split method
[Hahmann Bonneau-TVCG03]
Boundary Curves
Cross Derivatives
Inner Control Points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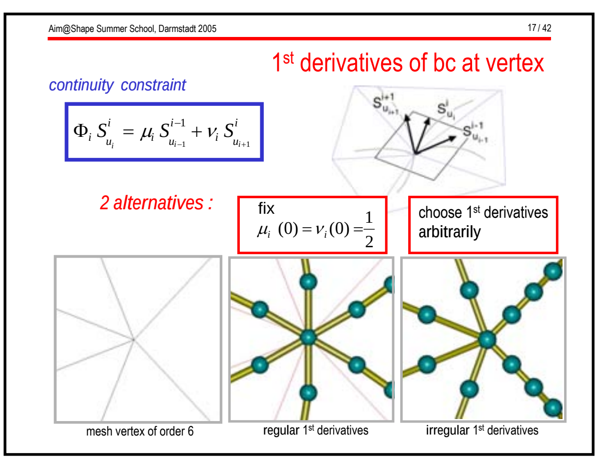
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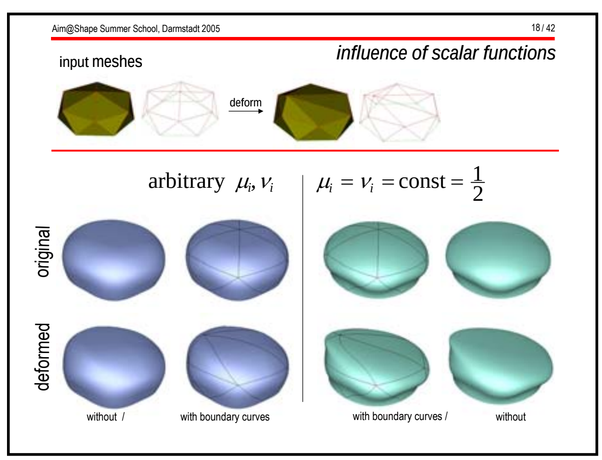
1st derivatives of bc at vertex
continuity constraint
$\Phi_i S^i_{u_i} = \mu_i S^{i-1}_{u_{i-1}} + \nu_i S^i_{u_{i+1}}$
2 alternatives :
fix $\mu_i(0) = \nu_i(0) = \frac{1}{2}$
choose 1st derivatives arbitrarily
mesh vertex of order 6
regular 1st derivatives
irregular 1st derivatives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influence of scalar functions
input meshes
deform
arbitrary μ_i, ν_i
$\mu_i = \nu_i = \text{const} = \frac{1}{2}$
original
deformed
without /
with boundary curves
with boundary curves /
without

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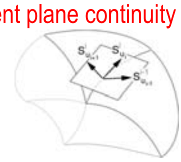
Tangent plane continuity

continuity constraint

$$\Phi_i S^i_{u_i} = \mu_i S^{i-1}_{u_{i-1}} + \nu_i S^i_{u_{i+1}}$$



twist constraint *(in matrix form)*

$$(M_3)T = (M_1)D_1 + (M_2)D_2$$

$$D_1^i = \frac{\partial S^i}{\partial u_i}(0,0)$$

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Vertex consistency problem

matrix (M_3) is rank deficient $\Leftrightarrow$ n is even

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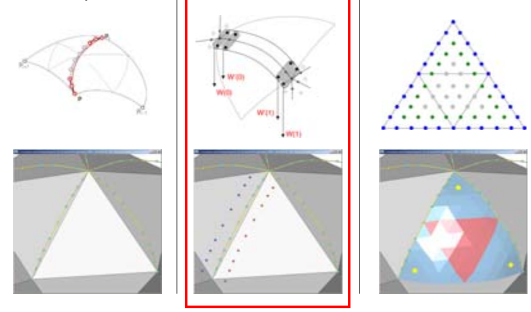
4-split method

[Hahmann Bonneau-TVCG03]

Boundary Curves | Cross Derivatives | Inner Control Points



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4-split method

[Hahmann Bonneau-TVCG03]

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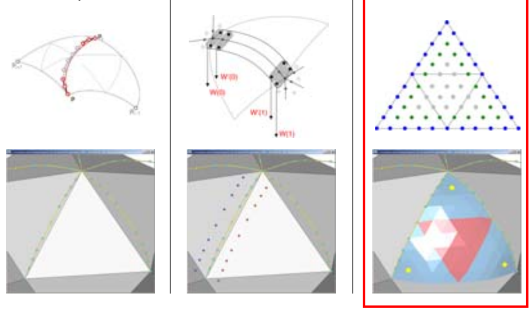

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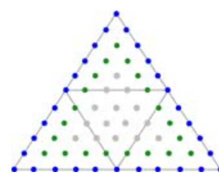
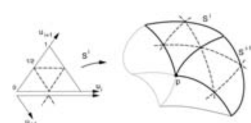
fill-in patches



G1 continuity

- boundary control points:
  piecewise degree 5 curves
- first row of control points :
  piecewise degree 4 cbt



domain 4-split

4 patches per macro-patch S^i

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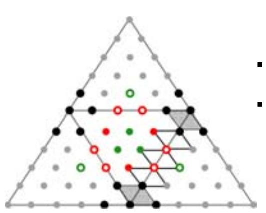
fill-in patches (cont.)



- edge mid-points are C1-continuous
- two triang. surfaces are C1 at common boundary ⇔ the three rows of cp form parallelograms

Make inner edges C1 continuous:

(1) choose ● => determines ○

(2) choose ● => determines ○

- 6 degrees of freedom

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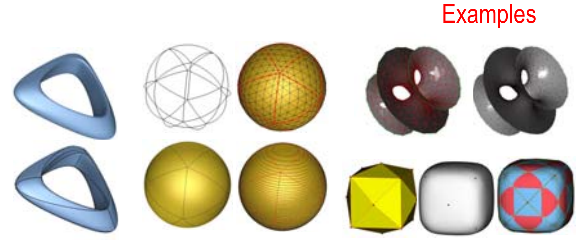
Free parameters

- Vertex positions (influences *n macro-patches*)
- First derivatives along boundary curves (influenecs *n macro-patches*)
- Twists (influences *n macro-patches*)
- 6 inner control points per macro-patch (influences *1 macro-patch*)

Examples



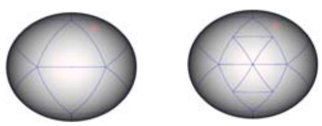
Hierarchical Modeling with Parametric Surfaces

Bibliography

- Hierarchical B-splines [Forsey, Bartels 88]
- Localized-Hierarchy surface splines [Gonzales, Peters 99]
- Hierarchical triangular splines [Yvart, Hahmann, Bonneau 05]

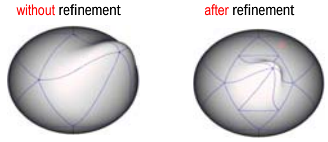
Why local refinement is needed ?

initial surface



Insertion of detail without refinement

Insertion of detail after refinement

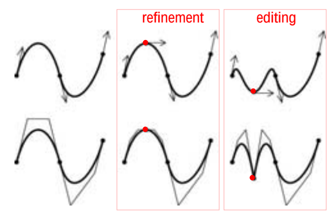


Curve refinement

Example of curves

Hermite spline

C1 Bézier curve



➔ Bézier subdivision is not sufficient for refinement

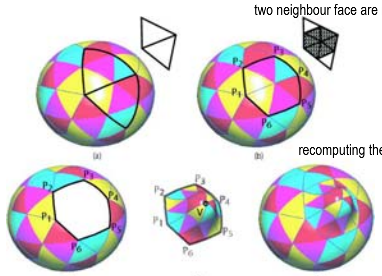
➔ a C1 interpolation scheme must be used

Local refinement

[Yvart Hahmann Bonneau-ACM ToG 05]

Principle of local surface refinement

two neighbour face are refined

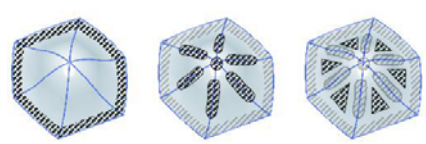


recomputing the surface

interpolation scheme is applied locally

Local refinement

Recomputing the finer patches

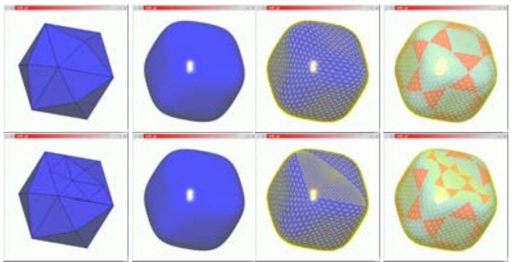


1. creation of G1 joint

2. solve vertex consistency
3. compute boundary curves
4. compute tangent planes

5. fill-in the macro-patches

Repeated local refinement



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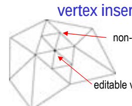
Hierarchical data structure

hierarchical surface modeling

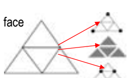
global modifications with few parameters
preservation of local details
LoD visualization

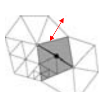
vertex insertion



hierarchical data structure

- *topological information* [Certain et al 96]



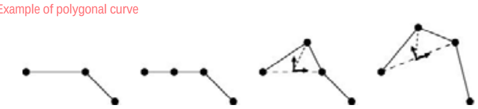


- *geometrical information*
  - origin & axes of local frame [Forsey/Bartels88]

  - local coordinates of free parameters
  - vertex
  - *n* 1st derivatives
  - *n* twists
  - 6 face control points for all faces around

Hierarchical Edition – Local Frames

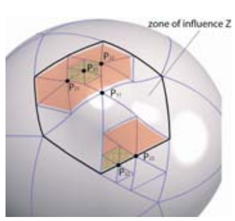
Example of polygonal curve



Each free parameter is encoded with respect to a local frame which belongs to the previous level.

$$\text{Coordinates}_{\text{Global}} = \text{Coordinates}_{\text{Reference}} + \text{Coordinates}_{\text{Displacement}}$$

Updating the hierarchy



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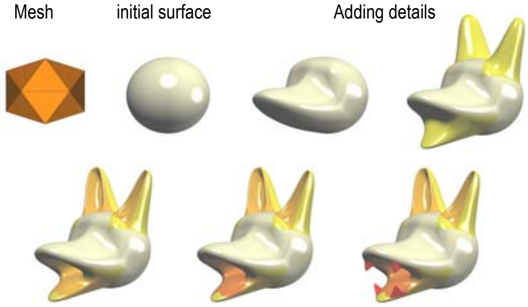
Hierarchical Modeling - Results

Construction of levels of detail

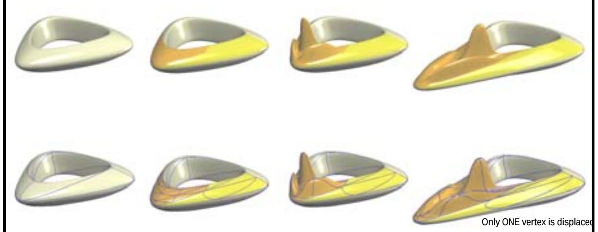
Mesh initial surface Adding details



Hierarchical Modeling - Results

Hierarchical Edition

Initial surface Refined surface Adding details Hierarchical edition



Hierarchical Modeling - Results

Hierarchical Edition

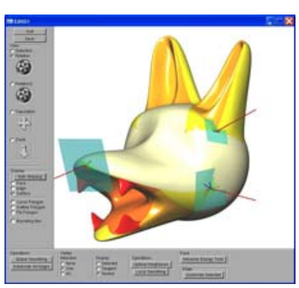
Only ONE vertex is displaced

Hierarchical Modeling - Results

Hierarchical Edition

Only ONE vertex is displaced

Application: Modeler



Application: Surface Fitting